

Carthage Mathematics Department
Course Summary for Math 3040: Abstract Algebra I

1. Credits: 4
2. Semesters Offered: Fall, Spring
3. Text(s): *Contemporary Abstract Algebra* (8th ed.), by Joseph Gallian
4. Topics Covered:
 - a. Properties of the integers
 - i. Divisibility
 - ii. Modular Arithmetic
 - iii. Equivalence relations
 - iv. Set maps (functions)
 - b. Elementary study of groups
 - i. Mathematical definition
 - ii. Dihedral groups
 - iii. Finite groups & subgroups
 - iv. Cyclic groups
 - v. Permutation groups
 - c. Theory of groups
 - i. Cosets & Lagrange's Theorem
 - ii. Orbits and Stabilizers
 - iii. Normal Subgroups & Quotient Groups
 - iv. Homomorphisms, Isomorphisms, and Automorphisms
 - v. The Isomorphism Theorems
 - vi. Direct Products
 - vii. Fundamental Theorem of Finite Abelian Groups
 - d. Elementary study of other algebraic structures
 - i. Introduction to rings
 - ii. Introduction to integral domains
 - iii. Introduction to fields
5. Skills Enhanced:
 - a. Proof writing: Students will regularly write mathematical proofs, both in homework assignments and in small collaborative groups. These will be handwritten, and typically not more than 1 page in length for each proof. The main goal of these assignments is for students to learn how to express mathematical ideas using the formality and rigorous logic that mathematics requires.
 - b. Oral presentation: Students will regularly present their work (from a variety of assignments) to the class. In addition to practicing presentation skills, this will permit students to critique each other's work, and participate in the critical inquiry that is crucial to the functioning of the mathematics community.
6. Sample Syllabus:

Parts 1 & 2 (Chapters 0-11), with selections from Part 3 (Chapters 12-13). Supplementary material can be taken from Part 5 (Chapters 24-33) as desired and appropriate.
7. Course Goals: By the end of the course, students should be able to do the following:
 - a. Differentiate between groups, rings, integral domains, and fields.
 - i. Assessment: Exams and/or quizzes include questions that require this knowledge.
 - ii. Assessment: Formal writing assignments require this knowledge.

- b. Analyze arithmetic structure of dihedral, modular, unit, and/or permutation groups by comparing properties of their respective elements
 - i. Assessment: Homework and writing assignments require this knowledge.
- c. State important definitions, including the definition of a group, LaGrange's Theorem, and the Fundamental Theorem of Finite Abelian Groups.
 - i. Assessment: Quiz questions will require this knowledge.
- d. Write induction, contradiction, and contrapositive proofs, using standard mathematical conventions.
 - i. Assessment: Proof-writing assignments require direct application of this knowledge.
- e. Demonstrate sufficient knowledge of the course content.
 - i. Assessment: Sufficient knowledge is required to obtain a passing grade. The knowledge must be demonstrated on exams, homework, and writing projects.